# Chattering adaptive control for first order systems

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*Abstract.-* A new fast switching adaptive algorithm is presented as an alternative solution to the Reference Model Adaptive Control (RMAC) problem for first order linear time-invariant systems. As shown through numeric examples, the proposed control scheme deals with the problem of parameter variance that arises when the nominal system presents un-modeled dynamics y added noise al the output of the plant to be controlled, that in many cases is destructive to the closed loop system. The closed loop Lyapunov stability analysis is presented.

**Key Words:** Adaptive Control, linear system, reference model.

*Resumen.-* Se presenta un nuevo algoritmo de adaptación por conmutación rápida como una solución alterna al problema de control adaptable por el método de modelo de referencia (CAMR) para sistemas lineales de primer orden invariantes en el tiempo. Como se muestra a través de experimentos numéricos, nuestra propuesta puede hacer frente al problema de deriva parámetrica que aparece cuando el sistema nominal presenta dinámicas no modeladas y ruido aditivo en la salida de la planta a controlar, que en muchos casos, esta deriva parámetrica resulta destructiva para el sistema en lazo cerrado. Análisis de estabilidad del sistema en lazo cerrado es hecho por medio de la teoría de estabilidad en el sentido de Lyapunov.

**Palabras clave:** Control adaptable, sistemas lineales, modelo de referencia.

## 1. Introduction

Reference Model Adaptive Control techniques have been widely used in linear systems, especially first order linear systems (Slotine & Li, 1999). This is because in engineering there are a number of systems of interest that can be modeled by first order systems. For instance, some thermal systems (Smith & Corripio, Chap. 3), the break system used automobiles and some electronic systems (Slotine & Li, 1999, Page 326) can be modeled by first order systems. One of the main problems of Reference Model Adaptive Control techniques is the parameter variance that that in most cases proves to be destructive to system to be controlled (Slotine & Li, 1999, Page 354). In this sense, as is shown through numeric examples, a new adaptation law is presented, that is capable of overcoming the problem of parametric variance. This proposal is based on the introduction of a fast switching term by means of the sign function into the control law, which makes the closed loop system more robust. The addition of fast switching terms has been effective in the control of robot manipulators with friction (Orlov et al., 2003). Lyapunov analysis is used to prove stability of the undisturbed closed loop system. This paper is distributed follows: Section II presents the modeling and solution of the reference model adaptive control giving a numeric example of the problem of parameter variance. Section III introduces the new algorithm and it is numerically shown that the parameter variance not present in section II. Finally, section IV presents the conclusions.

## 2. Reference Model Adaptive Control

Consider the nominal system to be approximately the following differential equation:

$$\dot{y} = -a_p y + b_p u \qquad (1$$

where $y$ is the plan output, $u$ is the control input and $a_p$ and $b_p$ are constant system parameters. From the adaptive control viewpoint, $a_p$ and $b_p$ are unknown, but the sign of $b_p$ ($sgn(b_p)$) is known.

Consider the following reference model:

$$\dot{y}_m = -a_m y_m + b_m r(t) \qquad (2)$$

where $a_m$ and $b_m$ are constant parameter and $r(t)$ is an external bounded reference input. The input parameter $a_m$ must be strictly positive to ensure the stability of the reference model.

*Outline of the RMAC problem.*

The reference model adaptive control problem consists of finding a control law $u(t)$ such that:

$$u(t) = \hat{a}_r(t)r + \hat{a}_y(t)y \qquad (3)$$

where $\hat{a}_r(t)$ and $\hat{a}_y(t)$ are adaptive gains that adjust dynamically, i. e., $\hat{a}_r(t)$ and $\hat{a}_y(t)$ are solutions to:

$$\dot{\hat{a}}_r(t) = f_1(\mathrm{sgn}(b_p), \gamma, e, r) \tag{4}$$

$$\dot{\hat{a}}_y(t) = f_2(\mathrm{sgn}(b_p), \gamma, e, y) \tag{5}$$

where $\gamma$ is a positive constant that represents the adaptation gain and $e=y-y_m$, so that:

$$A)\ \lim_{t\to\infty} e(t) = 0 \tag{6}$$

$$B)\ |\hat{a}_r(t)| < \infty \quad \text{and} \quad |\hat{a}_y(t)| < \infty \quad \forall t \geq 0 \tag{7}$$

*Commentary 1.* Under the assumption of persistent input, reference model adaptive control schemes can be used as parameter identification schemes, explicitly, the values of $a_p$ and $b_p$ may be estimated by $\hat{a}_r(t)$ and $\hat{a}_y(t)$ (Slotine & Li, 1999). ♦

Using:

$$\dot{\hat{a}}_r(t) = -\mathrm{sgn}(b_p)\gamma e r \tag{8}$$

$$\dot{\hat{a}}_y(t) = -\mathrm{sgn}(b_p)\gamma e y \tag{9}$$

solves the RMAC problem (Slotine & Li, 1999, Page 328). The problem of parameter variance may appear when the reference signal $r(t)$ is constant and the nominal model contains un-modeled dynamics and added external noise (Slotine & Li, 1999, Pages 354-356). Consider the example shown in Fig. 1 (Slotine y Li, 1999). By using $\gamma=1$, $r(t)=2$, $n(t)=0.5\sin(16.1t)$, $y_m(0)=0$, $y(0)=-1$, $\hat{a}_r(0)=1$, $\hat{a}_y(0)=-1$ and with the un-modeled dynamic at rest, the result shown in Fig. 2 are obtained. Note that the adaptation parameters $\hat{a}_r(t)$ and $\hat{a}_y(t)$ vary resulting in a variance of $y(t)$.

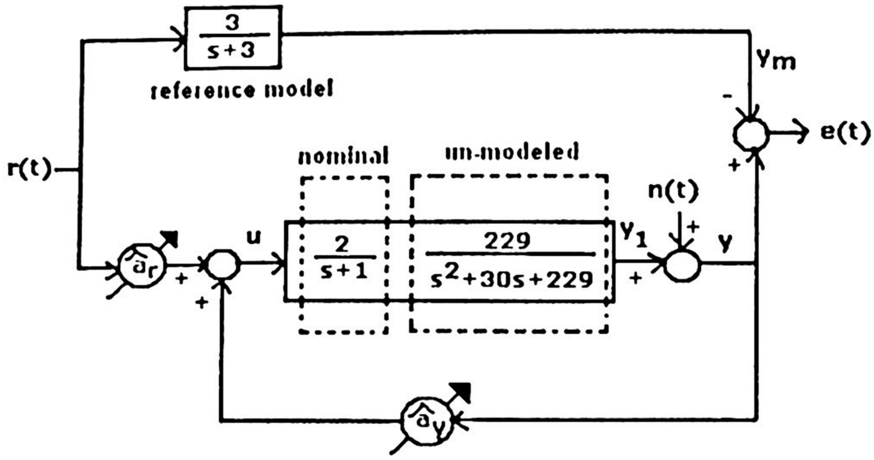


**Fig. 1** Adaptive control scheme with un-modeled dynamic and added noise.

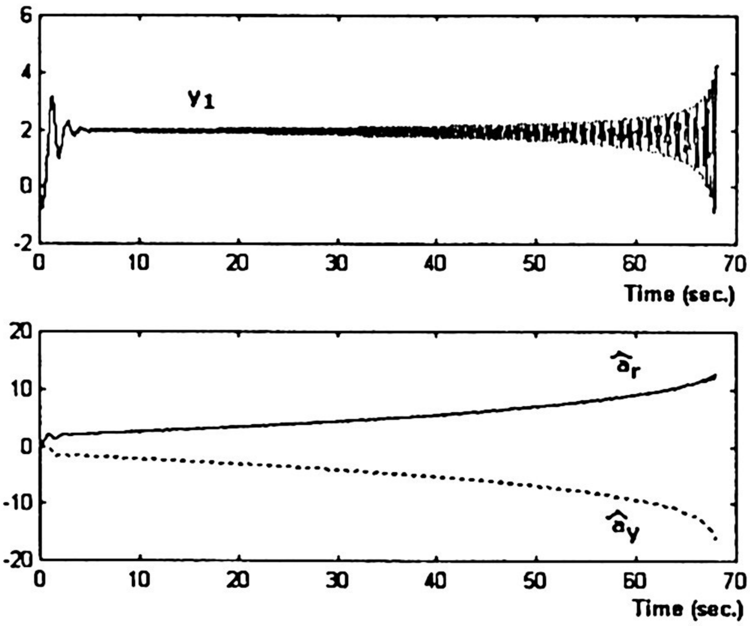


**Fig. 2** Parameter variance.

# 3. RMAC via fast switching

In (Orlov et al., 2003) it is shown that the proper introduction of fast switching robustifies the closed loop system. In this sense, the following solution to the RMA' problem is proposed:

*Theorem 1.* Given the nominal system (1) and the reference model (2) with $u(t)$ in (3)

$$\dot{\hat{a}}_r(t) = -\operatorname{sgn}(b_p)\gamma e r \tag{10}$$

$$\dot{\hat{a}}_y(t) = -\operatorname{sgn}(b_p)\gamma e \operatorname{sgn}(y) \tag{11}$$

the RMAC problem is solved.

The addition of the fast switching term is is shown in (11) through the use of the sign function.

*Proof.*
Let

$$\tilde{a}_r = \hat{a}_r - a_r^{\bullet} \tag{12}$$

$$\tilde{a}_y = \hat{a}_y - a_y^{\bullet} \tag{13}$$

where $a_r^{\bullet}$ and $a_y^{\bullet}$ are positive constants that would yield perfect model estimation, i.e., if

$$\hat{a}_r(t) = a_r^{\bullet} = \frac{b_m}{b_p} \tag{14}$$

$$\hat{a}_y(t) = a_y^{\bullet} = \frac{a_p - a_m}{b_p} \tag{15}$$

the closed loop system (3) in (1) would be:

$$\dot{y} = -a_m y + b_m r \tag{16}$$

For this, knowledge of $a_p$, and $b_p$ is obviously required. Given $e=y-y_m$, we obtain

$$\dot{e} = \dot{y} - \dot{y}_m \underset{(1),(2),\text{and}(3)}{=} -a_m(y - y_m) + (a_m - a_p + b_p\hat{a}_y)y + (b_p\hat{a}_r - b_m)r$$
$$= -a_m e + b_p(\tilde{a}_r r + \tilde{a}_y y) \tag{17}$$

Defining the Lyapunov function as:

$$V(e,\tilde{a}_r,\tilde{a}_y)=\frac{1}{2}e^2+\frac{1}{2\gamma}|b_p|\tilde{a}_r^2+\frac{k_1}{2\gamma}|b_p|\tilde{a}_y^2$$

where $k_I$ is a positive constant. The time derivative of (18) yields:

$$\dot{V}(e,\tilde{a}_r,\tilde{a}_y)=e\dot{e}+\frac{1}{\gamma}|b_p|\tilde{a}_r\dot{\hat{a}}_r+\frac{k_1}{\gamma}|b_p|\tilde{a}_y\dot{\hat{a}}_y$$

$$\underset{(10),(11),y(17)}{=}-a_m e^2+b_p\tilde{a}_r ye-k_1 b_p\tilde{a}_y e\,\mathrm{sgn}(y)$$

By using the following approximation:

$$\mathrm{sgn}(y)=\lim_{\varepsilon\to 0}\frac{y}{|y|+\varepsilon},$$

from (19) we have:

$$\dot{V}(e,\tilde{a}_r,\tilde{a}_y)=\lim_{\varepsilon\to 0}[-a_m e^2+b_p\tilde{a}_r ye-k_1 b_p\tilde{a}_y e(\frac{y}{|y|+\varepsilon})]-\frac{k_1 y}{|y|+\varepsilon}\leq -y$$

$$\Rightarrow |y|+\varepsilon\leq k_1$$

and thus from (20) we obtain:

$$\dot{V}(e,\tilde{a}_r,\tilde{a}_y)\leq[-a_m e^2+b_p\tilde{a}_r ye-b_p\tilde{a}_y ey]=-a_m e^2$$

or:

$$\dot{V}(e,\tilde{a}_r,\tilde{a}_y)\leq -a_m e^2 \quad (21$$

which implies that $e(t), \tilde{a}_r(t), \tilde{a}_y(t) \in L_\infty$, and from the integration of (21) we conclude that $e(t) \in L_2$. From the previous in (17) we conclude that $\dot{e}(t) \in L_\infty$ (since $y_m$ is bounded), and using Barbalat's lemma we conclude that $\lim_{t \to \infty} e(t) = 0$. ♦

To prove the effectiveness of the algorithm presented in *Theorem* 1, the same experiment shown in Fig. 1 is repeated with (10), (11) and (3). The results are shown in Fig. 3.

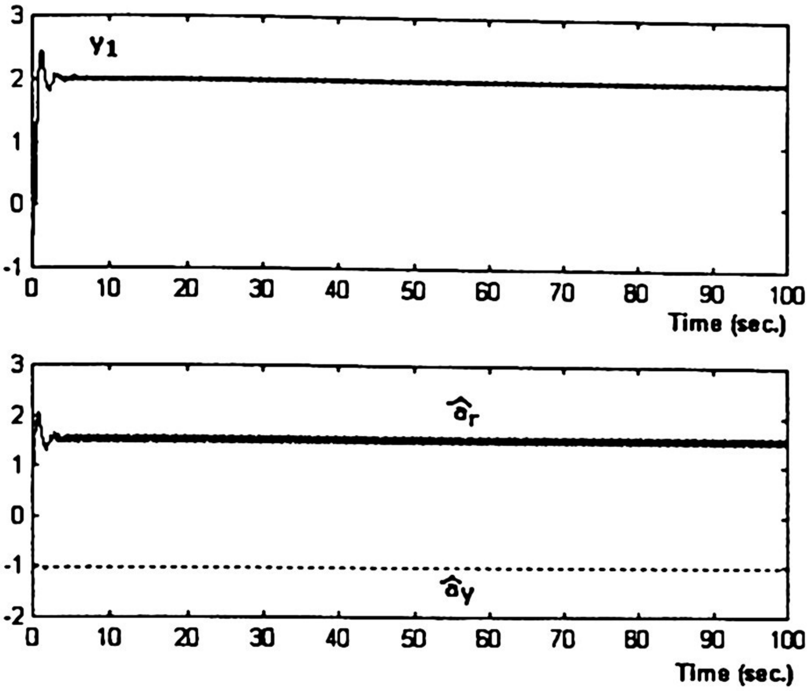


Fig. 3 Numeric results.

## 4. Conclusions

A new reference model adaptive control algorithm that uses a fast switching term in one of the adaptation laws, proving, from a numeric point of view, the efficacy of the algorithm when facing the problem of parametric variance. And while it is true that the inclusion of fast switching terms requires control signals that are rich in frequency content, said signals can be approximated by smoother functions (Christopher E., and Spurgeon S. K., 1998, Page 15), which is common in automatic control

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